

# MASSES AND WIDTHS OF THE $\rho^{\pm,0}(770)$

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## ABSTRACT

Isospin violation in the  $\rho(770)$  mass and width is considered within the  $S$  matrix approach using combined fits to the  $e^+e^- \rightarrow \pi^+\pi^-$  and  $\tau^- \rightarrow \nu_\tau\pi^-\pi^0$  data performed by the ALEPH collaboration. We show that the pole position following from the parameters obtained from the ALEPH fits are not sensitive to the details of the parametrization. In this context, we have found that the pole mass difference and the pole width difference between the charged and neutral  $\rho$  are consistent with zero. We show that a one loop calculation including vector, axial vector and pseudo-scalar mesons can satisfactorily describe the observed isospin breaking. We also give an estimate for the mass difference between the neutral and charged states of the  $a_1(1260)$ .

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## I. INTRODUCTION

Isospin is in very good approximation a symmetry of the strong interactions. At the fundamental level, isospin breaking occurs as a consequence of the electromagnetic interactions and the mass difference of the  $u$  and  $d$  quarks. The numerical value of the fine structure constant and the fact that  $|m_u - m_d|$  is small compared to  $\Lambda_{QCD}$  explains why this is a symmetry good at the few percent level. In this work we are concerned with the isospin and its breaking in the  $\rho$  resonances. The experimental tests of this symmetry requires the precise determination of the mass and width of the charged and neutral states, a non trivial task since a precise definition of these quantities must be adopted.

The values compiled by the Particle Data Group [1] for the  $\rho(770)$  mass and width are spread over a wide range. This is an unpleasant characteristic since one expects these intrinsic properties to be independent of the process where the resonance is observed and also to be model and background independent. This is related to the fact that different expressions are used to fit the data leading to different values for the parameters - that one is tempted to identify with the physical mass and width - which however are relevant only for the model under consideration. Although this is a practical way to deal with the  $\rho$  meson physics, *i.e.* use a given parametrization and the corresponding values for the mass and width, the process and model independence of the physical quantities associated to any resonance are relevant from the conceptual point of view.

The analysis of the experimental data [2–4] is based on theoretical assumptions such as a parametrization derived in terms of analyticity and unitarity [5], a Breit-Wigner which incorporates an energy dependent width reflecting the p-wave nature of the  $\rho$  and higher isovector-vector resonances [6] or still phenomenological effective field theories [7]. There is a priori no reason to favour any of these approaches in the description of physical process, on the contrary one would expect them to yield complementary or at least consistent information. All of them share two characteristics: *i)* they involve a momentum dependent width  $\tilde{\Gamma}(s)$ , and *ii)* they define the resonance mass  $\tilde{M}$  as the energy for which the real part of the propagator is zero and the width as  $\tilde{\Gamma}(s = \tilde{M}^2)$ . We will refer collectively to them as the *conventional*, as opposed to the *pole* approach to be discussed next. On the other hand, the  $S$  matrix formalism [8] relates the mass and width of the resonance to pole(s) of the amplitude in the complex plane. The pole associated to a given resonance is independent of the process where it is observed [8,9], thus this scheme offers the possibility to get an unambiguous determination of the intrinsic properties of the resonance. For the  $\rho(770)$  this has been done in [10].

The  $S$  matrix formalism not only provides a convenient parametrization in terms of which the electromagnetic form factor of the pion can be expressed, but it can also be applied to any of results of the conventional approach. In fact we have two possibilities: the first is to make an expansion around the pole position of the expressions used in the conventional approach and then fit the data. We already know the result of such a procedure since in all cases the form factor will reduce to the  $S$  matrix formalism, and the fit using this scheme was performed in [10]. Alternatively, fits to the data can be carried using the conventional parametrization and then, given the expression used and the parameters obtained in fitting the data, one can determine the associated pole. It is our purpose in this letter to perform the latter analysis considering fits reported for  $e^+e^- \rightarrow \pi^+\pi^-$  and for the tau decay  $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$  which allow us to make a statement about the size of isospin violation in the  $\rho^\pm - \rho^0$  system within a framework in which the masses and widths are unambiguously defined.

We also consider the problem from the theoretical point of view. To this end we take into account both possible sources of isospin violation: the  $m_u - m_d$  mass difference and the electromagnetic interactions, which is implemented at the hadron level by the use of vector meson dominance. The former contribution is known to vanish to lowest order implying thus a finite result for the latter. The explicit calculation of the one loop electromagnetic self energy shows that a finite result can be achieved only when the anomalous magnetic moment  $\kappa$  entering the three  $\rho$  vector meson vertex (Vector Meson Dominated  $\rho - \rho - \gamma$  vertex) takes the value  $\kappa = 1$ . We briefly comment on this point.

## II. M AND $\Gamma$ IN THE CONVENTIONAL AND POLE APPROACHES.

Within the **conventional approach**, near the  $\rho$  resonance, the electromagnetic form factor of the pion is parametrized in terms of a Breit-Wigner with a momentum dependent width:

$$F_\pi(s) = \frac{A(s)}{D(s)} ; \quad D(s) = s - \tilde{M}_\rho^2(s) + i\tilde{M}_\rho(s)\tilde{\Gamma}_\rho(s) \quad (1)$$

Although some expressions occurring in the literature seem to differ from Eq.(1) they can be cast in that form by an appropriated redefinition of  $A(s)$  and  $\tilde{\Gamma}(s)$ . For example if instead of  $A(s)$ , we use  $\bar{A}(s) = A(s)(1 - ix(s))$ ,  $D(s)$  should be changed to:

$$\bar{D}(s) = s - \tilde{M}_\rho^2(s) + \tilde{M}_\rho(s)\Gamma_\rho(s)x(s) + i(\tilde{M}_\rho(s)\tilde{\Gamma}_\rho(s) - (s - \tilde{M}_\rho^2(s))x(s)). \quad (2)$$

Since in the conventional approach the mass and width of the resonance are defined respectively by

$$Re(D(m_\rho^2)) = 0 ; \quad \text{and} \quad Im(D(m_\rho^2)) = m_\rho\Gamma_\rho, \quad (3)$$

the freedom in the choice of parametrization (compare Eqs.(1,2)) implies an arbitrariness in the mass and width definition. On the other hand, the ***S matrix approach*** [8,9] avoids this problem by considering a simple observable pole with  $s$  independent width and residue:

$$F_\pi(s) = \frac{R_p}{s - s_p} + B(s), \quad (4)$$

where  $s_p$  is the pole position,  $R_p$  the pole residue and  $B(s)$  is the remaining part around the pole. In order that this description makes sense the remaining part  $B(s)$  (which is fixed by the fit to the data) should be a soft function of  $s$  affecting minimally the pole position, except for obvious resonance effects as in  $\rho - \omega$  mixing [10]. In this formalism the mass  $M$  and width  $\Gamma$  of the resonance are defined through:

$$s_p = M^2 - iM\Gamma. \quad (5)$$

The fact that the background is a soft function around the pole (in this case for  $\sqrt{s} \approx m_\rho$ ), does not imply that it can not have poles at other energies, however these can be included in an appropriated way in the background. Different parametrizations within the conventional approach correspond to different backgrounds in the  $S$  matrix formalism. Thus even though the mass and width values obtained from a given “conventional” treatment are only relevant for that particular parametrization, on physical grounds one expects that all of them have a pole in the same position providing thus, through Eq.(5), the physical mass and width of the resonance. Notice that in finding the pole  $D(s_p) = 0$  no information about the overall normalization nor about  $A(s)$  is required, as one would expect from the  $S$  matrix approach.

The cleanest determination of the  $\rho$  mass and width comes from the  $e^+e^-$  annihilation and tau-lepton decay data, which are in agreement with each other up to an overall normalization factor [1]. Although several analysis for the  $e^+e^- \rightarrow \pi^+\pi^-$  [2,4,7,11,12] and  $\tau^- \rightarrow \nu_\tau\pi^-\pi^0$  [2,3] data have been performed, we concentrate in the results of the ALEPH collaboration [2] which seems to us the best suited for a study of isospin breaking since a combined fit to both sets of data was carried by these authors. Notice however that the agreement is excellent with the fit to the data of the high statistics experiment by the CLEO collaboration [3]. The  $\rho^\pm$ ,  $\rho^0$  masses and widths resulting from the analysis in [2] and the values associated to the corresponding pole position are summarized in Table 1 for the masses and Table 2 for the widths.

Fit	$m_{\rho^\pm}(MeV)$		$m_{\rho^0}(MeV)$		$m_{\rho^\pm} - m_{\rho^0}(MeV)$	
	conven.	pole	conven.	pole	conven	pole
KS( $\lambda = 1$ )	$773.4 \pm 0.9$	$757.0 \pm 1.3$	$773.4 \pm 0.7$	$756.9 \pm 1.0$	$0.0 \pm 1.0$	$0.1 \pm 1.6$
GS( $\lambda = 1$ )	$775.7 \pm 0.9$	$758.1 \pm 1.3$	$775.7 \pm 0.7$	$758.0 \pm 1.0$	$0.0 \pm 1.0$	$0.1 \pm 1.6$
GS( $\lambda = 0.45 \pm 0.11$ )	$783.8 \pm 3.0$	$758.3 \pm 5.4$	$783.8 \pm 3.0$	$758.1 \pm 5.4$	$0.0 \pm 1.2$	$0.2 \pm 7.6$

Table 1. Masses of the  $\rho^\pm$  and  $\rho^0$  resonances as obtained by Barate *et al* [2] (second and fourth column) from a combined fit to  $e^+e^- \rightarrow \pi^+\pi^-$  and  $\tau^- \rightarrow \nu_\tau\pi^-\pi^0$  data. For comparison in the third and fifth column we quote the values associated to the corresponding pole. Details about the notation can be found in the quoted literature.

Fit	$\Gamma_{\rho^\pm}(MeV)$		$\Gamma_{\rho^0}(MeV)$		$\Gamma_{\rho^\pm} - \Gamma_{\rho^0}(MeV)$	
	conven.	pole	conven.	pole	conven	pole
KS( $\lambda = 1$ )	$147.7 \pm 1.6$	$143.2 \pm 1.5$	$147.3 \pm 1.3$	$142.7 \pm 1.2$	$0.4 \pm 1.0$	$0.5 \pm 1.9$
GS( $\lambda = 1$ )	$150.8 \pm 1.7$	$145.3 \pm 1.5$	$150.8 \pm 1.3$	$145.2 \pm 1.2$	$0.0 \pm 2.0$	$0.1 \pm 1.9$
GS( $\lambda = 0.45 \pm 0.11$ )	$162.0 \pm 5.3$	$145.1 \pm 6.3$	$162.4 \pm 5.0$	$145.2 \pm 6.5$	$-0.4 \pm 2.5$	$-0.1 \pm 8.7$

Table 2. Widths of the  $\rho^\pm$  and  $\rho^0$  resonances as obtained by Barate *et al* [2] (second and fourth column) from a combined fit to  $e^+e^- \rightarrow \pi^+\pi^-$  and  $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$  data. For comparison in the third and fifth column we quote the values associated to the corresponding pole. Details about the notation can be found in the quoted literature.

The results are remarkable. While in the conventional approach the masses are spread over a 10 MeV range, the  $S$  matrix formalism leads values that differ at most in 1.3 MeV. Similarly for the width where the two approaches yield values differing, within each scheme, in 14 and 2 MeVs respectively. We can safely conclude that in contradistinction to the conventional approach the pole position is independent of the details of the parametrization used to fit the data providing thus an unambiguous definition for the mass and width of the  $\rho$ . It is worth remarking that the mass and width values we obtained applying the  $S$  matrix formalism to different parametrizations of the conventional approach agree with previous analysis performed fully in the  $S$  matrix formalism [10,13–15] but are systematically below the values obtained in the conventional approach and the value reported by the PDG [1].

As far as the isospin breaking is concerned, both schemes lead similar results. Although in the conventional approach there is a strong dependence upon the parametrization used to fit the data, it is the same for the charged and neutral channels and it cancels out when differences are considered. Let us emphasize again that the advantage of the pole approach is that it is based on an unambiguous definition of the mass and width and so, besides a nearly vanishing  $\rho^\pm - \rho^0$  mass difference we have a precise knowledge of each mass and width. The isospin violation in the coupling constants,  $g_\rho$ , implied by the pole decay rates is also consistent with zero,  $g_{\rho^\pm}^2$  and  $g_{\rho^0}^2$  are equal up to one percent.

When applying the  $S$  matrix formalism to one of the parametrizations used in the conventional approach, as we have done, the uncertainty in the pole position follow from the result of the original fit. In order to calculate the errors quoted in Tables 1 and 2, we used the following procedure: we obtain the pole position using the central value of the parameters obtained from the fit, we then determine the variation of the pole position when the parameters of the fit are changed by one standard deviation. We quote for the pole position error the maximum variation found in this way. One further comment regards the  $\lambda$  parameter [12] used in the second version of the Gounaris-Sakurai parametrization [2].  $\lambda$  modify the  $s$  dependence of the  $\rho$  decay width and it also changes the analytical structure obtained in an effective field theory. Notice that the errors are much larger when  $\lambda$  is left as a free parameter, *i.e.* when the  $s$  dependence is not the one predicted by the effective field theory [7].

### III. $M_{\rho^\pm} - M_{\rho^0}$ MASS DIFFERENCE WITHIN A VECTOR MESON DOMINANCE MODEL.

In this section we consider the theoretical evaluation of the  $\rho^\pm - \rho^0$  mass difference. In order to calculate the contribution due to the  $m_u - m_d$  mass difference, we need to evaluate the matrix element of the flavour symmetry breaking Hamiltonian  $H_B = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s$ . The matrix elements between  $\rho$  states can be written as:

$$\begin{aligned}
\langle \rho | (m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s) | \rho \rangle &= \frac{1}{\sqrt{2}}(m_u - m_d) \langle \rho | \frac{\bar{u}u - \bar{d}d}{\sqrt{2}} | \rho \rangle \\
&+ \frac{1}{\sqrt{6}}(m_u + m_d - 2m_s) \langle \rho | \frac{\bar{u}u + \bar{d}d - 2\bar{s}s}{\sqrt{6}} | \rho \rangle + \frac{1}{\sqrt{3}}(m_u + m_d + m_s) \langle \rho | \frac{\bar{u}u + \bar{d}d + \bar{s}s}{\sqrt{3}} | \rho \rangle.
\end{aligned} \tag{6}$$

The operators entering in the two last matrix elements are singlet under  $SU(2)$ , therefore they give the same contribution to the  $\rho^\pm$  and  $\rho^0$  and thus, do not contribute to the mass difference. On the other hand the first matrix element vanishes identical as consequence of  $G$  parity. We therefore conclude that to lowest order in  $(m_u - m_d)$  the  $\rho^\pm - \rho^0$  mass difference vanishes:

$$m_{\rho^\pm} - m_{\rho^0}|_{m_u \neq m_d} = 0. \quad (7)$$

This result has an important implication. Since the  $m_{u,d}$  quark masses have completely disappeared from our calculations, there are no fundamental constants of the underlying QCD theory which could be used to renormalize, *i.e.* to absorb possible divergencies appearing in the calculation. Therefore the electromagnetic self-mass contribution must be finite [16]. Later on we will see that the same conclusion can and has been reached [17] using different arguments.

Let us now consider the second source of isospin breaking, the electromagnetic interactions. The lowest order contributions are shown in Fig. 1(a – e), where  $P$  stands for a pseudoscalar meson which in our case can be either  $\pi$ ,  $\eta$  or  $\eta'$ ,  $a_1$  denotes an axial vector and  $V = \rho, \omega$  or  $\phi$  enters through Vector Meson Dominance. The first diagram is non vanishing for the charged  $\rho$  and vanishes for the  $\rho^0$ . Diagram  $b$  is non vanishing ( $V = \omega$  or  $\phi$ ) both for the charged and neutral contribution. Notice furthermore that the  $\eta$  or  $\eta'$  can be intermediate state only for the neutral  $\rho^0$  self-energy. We will neglect the contribution of the  $\eta$  and  $\eta'$  as we expect these to be small (one hundred times smaller than the contribution of diagram  $e$ ). On the other hand, the contribution from the loop with an internal pion yields the same contribution for the charged and neutral  $\rho$ 's. The third diagram vanishes identically when  $V = \rho$  due to  $G$  parity and gives the same contribution to the  $\rho^\pm$  and  $\rho^0$  self-mass when  $V = \omega$  or  $\phi$  and therefore does not contribute to the mass difference. Figures  $d$  and  $e$  are non-vanishing and contribute only to the charged and neutral meson mass respectively. Thus, in the model we are considering, the  $\rho^\pm - \rho^0$  mass difference is given by the contributions shown in Fig. 1 (a, d and e). To describe the photon  $\rho$  vertex we use Vector Meson Dominance, so that the  $\rho$  form factor is modeled by a  $\rho$  propagator. The Feynman rules for the electromagnetic interaction of a charged massive spin  $S = 1$  vector meson can be found in [18]. These rules depend upon the anomalous magnetic moment  $\kappa$  of the vector meson. Using these rules, we find that the correction to the  $\rho^\pm$  mass is given by (for later use we introduce obvious notation for the loop integrals of  $T_1, T_2$  and  $T_3 k_\alpha k_\beta$ ):

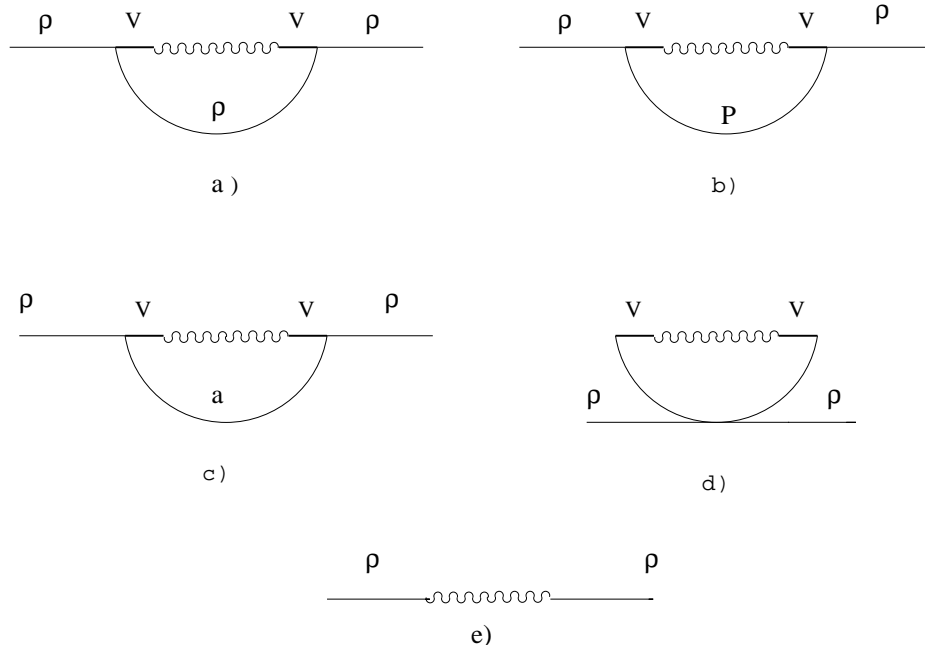


Fig. 1

One loop electromagnetic contribution to the self-mass of the  $\rho$ . Vector Meson Dominance is used to describe the coupling of the photon to the hadrons.  $P$  stands for a pseudoscalar ( $\pi$ ,  $\eta$  or  $\eta'$ ),  $V$  for a vector meson ( $\rho$ ,  $\omega$  or  $\phi$ ) and  $a$  for the axial vector  $a_1(1260)$ . See main text for further details.

$$\begin{aligned}
i\delta_{m_{\rho^\pm}^2}^\gamma &= -e^2 M^4 \epsilon_i^\alpha(p) \epsilon_i^{*\beta}(p) \int \frac{d^4 k}{(2\pi)^4} (T_1 g_{\alpha\beta} + T_2 g_{\alpha\beta} + T_3 k_\alpha k_\beta) \\
&= \frac{\alpha M^4}{4\pi} (t_1 + t_2 + t_3)
\end{aligned} \tag{8}$$

where:

$$\begin{aligned}
T_1 &= \frac{4m^2}{k^2 [k^2 - M^2]^2 [k^2 + 2pk]} - \frac{2}{k^2 [k^2 - M^2]^2} - \frac{1}{[k^2 - M^2]^2 [k^2 + 2pk]}, \\
T_2 &= \frac{\kappa^2}{[k^2 - M^2]^2 [k^2 + 2pk]} - \frac{\kappa^2 k^2}{4m^2 [k^2 - M^2]^2 [k^2 + 2pk]} - \frac{3\kappa^2 - 8\kappa + 4}{4m^2 [k^2 - M^2]^2}, \\
T_3 &= \frac{\kappa^2 + 4\kappa - 4}{k^2 [k^2 - M^2]^2 [k^2 + 2pk]} + \frac{\kappa^2 - 2\kappa + 1}{m^2 [k^2 - M^2]^2 [k^2 + 2pk]}.
\end{aligned} \tag{9}$$

Remark that the  $T_1$  contribution to the integral is exactly the same entering in the calculation of the electromagnetic mass difference of the pion [16]. Notice furthermore that as it stands, the expression involves ultraviolet divergences. For completeness we quote the analytical expression for  $\delta_{m_\rho}^\gamma$  as a function of  $\kappa$  and the  $\rho^\pm(m)$  and  $\rho^0(M)$  masses:

$$\begin{aligned}
t_1 &= \frac{2}{M^2} + \frac{1}{2m^2} \ln \frac{M^2}{m^2} + \frac{M^2 + 2m^2}{m^2 M^4} R \tan^{-1}\left(\frac{R}{M^2}\right), \\
t_2 &= -\frac{\Delta}{m^2} (\kappa^2 - 2\kappa + 1) - \frac{\kappa^2}{4m^2} \left[ 2 + \left(3 - \frac{M^2}{m^2}\right) \ln \frac{M^2}{m^2} + \frac{2(M^2 - 4m^2)(M^2 - m^2)}{m^2 R} \tan^{-1}\left(\frac{R}{M^2}\right) \right], \\
t_3 &= \frac{(\kappa^2 - 2\kappa + 1)}{4m^2} \left[ 3 + \Delta + \frac{M^4 - 4m^2 M^2 + 2m^4}{2m^4} \ln \frac{M^2}{m^2} - \frac{M^2}{m^2} - \frac{2m^2 - M^2}{M^4} R \tan^{-1}\left(\frac{R}{M^2}\right) \right] \\
&\quad + \frac{(\kappa^2 + 4\kappa - 4)}{4m^2} \left[ -\frac{2}{3} + \frac{M^2 - 3m^2}{3m^2} \ln \frac{M^2}{m^2} - \frac{2(m^2 - M^2)(4m^2 - M^2)}{3M^2 R} \tan^{-1}\left(\frac{R}{M^2}\right) \right].
\end{aligned} \tag{10}$$

with:

$$\Delta = \frac{2}{4-n} - \gamma + \ln(4\pi) - \ln \frac{M^2}{\mu^2} \quad R = \sqrt{M^2(4m^2 - M^2)}. \tag{11}$$

The self-energy involves ultraviolet divergencies - contained in the  $\Delta$  factors - which are absent for  $\kappa = 1$ . On the other hand the finiteness of the  $\rho$  electromagnetic self-mass is required not only from the reasoning presented paragraphs above. In Ref. [17] the same conclusion was reached using completely different arguments (based on dispersion relations and Regge pole theory). Furthermore one expects the  $\rho - \rho - \rho$  vertex to be symmetric in the indices, which is achieved only if  $\kappa = 1$ . Taking that value for  $\kappa$  and  $m = M = 757.5 \text{ MeV}$  we get:

$$\delta_{m_{\rho^\pm}^2}^\gamma = \frac{\delta_{m_{\rho^\pm}^2}^\gamma}{2m_\rho} = \frac{\alpha m_\rho}{8\pi} \left\{ 2 + \pi\sqrt{3} - \frac{2}{3} \right\} = 1.49 \text{ MeV}. \tag{12}$$

On the other hand, in the model we are considering the correction to the  $\rho^0$  meson mass is given by the diagramm in Fig.1(e). which leads to:

$$\delta_{m_\rho}^\gamma = \frac{\delta_{m_{\rho^0}^2}^\gamma}{2m_{\rho^0}} = \frac{2\pi\alpha m_{\rho^0}}{f_\rho^2} \quad (13)$$

With  $4.83 < f_\rho < 4.91$  (as determined in Ref. ([13])) from fits to  $e^+e^- \rightarrow \pi^+\pi^-$  data in the  $\rho$  region in the framework of the  $S$  matrix formalism, we use the central value  $f_\rho = 4.87$  so that putting all together we finally obtain:

$$\Delta m_\rho = m_{\rho^\pm} - m_{\rho^0} = \delta_{m_{\rho^\pm}} - \delta_{m_{\rho^0}} = 0.02 \pm 0.02. \quad (14)$$

Our result is consistent with a previous calculations by Bijmens and Gosdzinsky [19] who using a different approach find  $-0.7MeV < m_{\rho^\pm} - m_{\rho^0} < 0.4MeV$ .

A similar analysis can be carried for the  $a_1(1260)$ . To obtain an estimate we neglect the analog of diagram in Fig. 1b which contributes to the self mass of the charged but not of the neutral  $a_1(1260)$ .  $G$  parity requires  $V = \rho$  and the direct coupling of the photon to the neutral  $a_1$  (Fig. 1e) is not allowed. The contribution to the  $a_1(1260)$  mass difference from diagrams 1a and 1d is obtained replacing  $m_\rho$  by  $m_a$  in Eq. (12), which leads to:

$$\Delta m_a = \delta m_{a^\pm} - \delta m_{a^0} = \delta m_{a^\pm} = 2.42MeV. \quad (15)$$

#### IV. SUMMARY

In this work we considered the difference in masses and widths of the charged and neutral  $\rho(770)$  mesons. The  $S$  matrix formalism provides a framework where these intrinsic properties are unambiguously defined. The determination of the mass and width of the  $\rho$  in that framework have been presented in Ref. [10,13]. Here we go one step further by taking as starting point the combined fit to the  $e^+e^- \rightarrow \pi^+\pi^-$  and  $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$  data performed by the ALEPH collaboration [2]. The anlysis in [2] was carried using the Khun-Santamaria [6] formalism and two versions of the Gounaris-Sakurai [5] parametrization of the pion form factor. We have shown that the pole position following from the parameters obtained from the fit to the data are not sensitive to the details of the parametrization. In this context, we have found that the mass difference between the charged and neutral  $\rho$  is consistent with zero very much as the width is. Our results for the  $\rho$  mass and width agree with the pole position found when the pion form factor is parametrized from the very begining according to the  $S$  matrix formalism and are smaller than the values reported by the PDG.

From the theoretical point of view, we considered the isospin violation induced by the  $m_u - m_d$  mass difference which is known to vanish to the lowest order. We have argued that the electromagnetic self-mass should be finite and we have shown that this result is obtained when the electromagnetic coupling of the  $\rho$  is modeled using vector meson dominance and the  $\rho$  anomalous magnetic moment  $\kappa$  takes the value  $\kappa = 1$ . The model satisfactorily describes the observed isospin breaking.

Let us finally comment that the parametrization independence of the pole position holds when a larger set of fits is considered. Indeed when the typical parametrizations used in Ref. [2–4,7,12] are analyzed along the lines of this work (except for the fit called VMD2(NU) [7], which as pointed out by the author leads to a  $\chi^2 = 148/77$  compared to the  $\chi^2 \approx 1$  of all the others fits) the mass values associated to the pole are spread over a five MeV region around 758 MeV, in contrast to the 25 MeV around 767 MeV obtained in the conventional approach. A similar situation holds for the width where the spreads are respectively 7 and 22 MeV and the central values 143 and 151 MeV respectively.

## V. ACKNOWLEDGMENTS

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